MAT 2384-Practice Problems on Interpolation Methods

1. Calculate the **Lagrange polynomial** of degree 2, $p_2(x)$, to 4 decimal places that fits the following three data points $(x_i,\, f(x_i))$ for a certain unknown function f:

$$(1.01,\, 1), \quad (1.02,\, 0.9888), \quad (1.04,\, 0.9784)$$

and from it interpolate a value of f at $x = 1.035$ and extrapolate a value of f at $x = 1.055$. Give error bounds on your estimation of $f(1.035)$ if $0.251 \leq f'''(t) \leq 0.45$ for any $t \in [1,\, 1.04]$.

2. The error function is given by $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2} dt$. Note that it hard to get an exact value for $\text{erf}(x)$ since we don't know an antiderivative for e^{-t^2}. Given that $\text{erf}(0.25) = 0.27633$, $\text{erf}(0.5) = 0.52050$ and $\text{erf}(1) = 0.84270$, calculate the **Lagrange polynomial** of degree 2, $p_2(x)$, to 5 decimal places that approximates $\text{erf}(x)$. Use it to approximate $\text{erf}(0.75)$. Using the error formula for Lagrange Interpolation, give bounds on your estimation of $\text{erf}(0.75)$. [Hint. The following fact from Calculus is useful: If $f(x) = \int_a^x g(t)dt$, then $f'(x) = g(x)$]

3. Given that $f(0) = 0$, $f(1) = 0.9461$, $f(2) = 1.6054$, use **Newton's forward difference formula** to find $p_2(x)$ and use it to estimate the value at $x = 1.5$.

4. Given that $f(0.5) = 0.479$, $f(1) = 0.841$ and $f(2) = 0.909$ for some unknown function f, estimate $f(0.8)$ and $f(0.9)$ by quadratic interpolation via Newton's divided difference polynomial (with coefficients rounded to 5 decimal places).

5. Given the four data points $(x_i,\, f(x_i))$ from an unknown function f:

$$(1,\, -3.02), \quad (2,\, 1.25), \quad (3,\, 3.1487), \quad (4,\, -2.546)\,:$$

   (a) estimate $f(2.5)$ and $f(3.5)$ by cubic interpolation via Newton's divided difference polynomial (with coefficients rounded to 5 decimal places).

   (b) estimate $f(2.5)$ and $f(3.5)$ by cubic interpolation via Newton's forward difference formula.

   (c) Given that $1 \leq |f^{(4)}(t)| \leq 2$ for any $t \in [1,\, 4]$ give error bounds for your estimates in part (a).